Pochodne

przykładowe zadania na kartkówkę

1. $f(x) = \sqrt[3]{x} \cdot \sin\left(\frac{x^3+1}{2x+\ln(5x)}\right)$

2. $f(x) = \cos\left(\operatorname{arctg}\left(\sqrt{\frac{5}{x^2}+\ln^3 x}\right)\right)$

3. $f(x) = \left(2^{\sqrt{x}} + 5\,\operatorname{tg}^2 x\right)^{\sin x}$

Rozwiązanie.

Ad 1.

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \cdot \sin\left(\frac{x^3+1}{2x+\ln(5x)}\right) +$$

$$+ \sqrt[3]{x} \cdot \left[\cos\left(\frac{x^3+1}{2x+\ln(5x)}\right)\right] \cdot \frac{3x^2 \cdot (2x+\ln(5x)) - (x^3+1)\cdot\left(2+\frac{1}{5x}\cdot 5\right.}{(2x+\ln(5x))^2}$$

Ad 2.

$$f'(x) = -\left[\sin\left(\operatorname{arctg}\left(\sqrt{\frac{5}{x^2} + \ln^3 x}\right)\right)\right] \cdot$$

$$\cdot \frac{1}{1 + \left(\sqrt{\frac{5}{x^2} + \ln^3 x}\right)^2} \cdot$$

$$\cdot \frac{1}{2\sqrt{\frac{5}{x^2} + \ln^3 x}} \cdot \left(\frac{-10}{x^3} + 3(\ln^2 x) \cdot \frac{1}{x}\right)$$

Ad 3.

$$f(x) = \left(2^{\sqrt{x}} + 5\operatorname{tg}^2 x\right)^{\sin x} \quad \Big| \ln(\ldots)$$

$$\ln(f(x)) = \ln\left(2^{\sqrt{x}} + 5\operatorname{tg}^2 x\right)^{\sin x}$$

$$\ln(f(x)) = \sin x \cdot \ln\left(2^{\sqrt{x}} + 5\operatorname{tg}^2 x\right) \quad \Big| (\ldots)'$$

$$\left(\ln(f(x))\right)' = \left(\sin x \cdot \ln\left(2^{\sqrt{x}} + 5\operatorname{tg}^2 x\right)\right)'$$

ILOCZYN $\rightarrow (fg)' = f'g + fg'$

$$\frac{1}{f(x)} \cdot f'(x) = (\sin x)' \cdot \ln\left(2^{\sqrt{x}} + 5\operatorname{tg}^2 x\right) + (\sin x) \cdot \left(\ln\left(2^{\sqrt{x}} + 5\operatorname{tg}^2 x\right)\right)'$$

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$$\frac{f'(x)}{f(x)} = (\cos x)\cdot \ln(2^{\sqrt{x}} + 5tg^2x) +$$

$$+ (\sin x)\cdot \frac{1}{2^{\sqrt{x}} + 5tg^2x}\cdot(2^{\sqrt{x}} + 5tg^2x)'$$

$$(a^{f(x)})' = a^{f(x)}\cdot(\ln a)\cdot f'(x)$$

$$tg^2x = (tgx)^2$$

$$((tgx)^2)' = 2(tgx)\cdot(tgx)' = \ldots$$

$$\frac{f'(x)}{f(x)} = (\cos x)\cdot \ln(2^{\sqrt{x}} + 5tg^2x) +$$

$$+ (\sin x)\cdot \frac{1}{2^{\sqrt{x}} + 5tg^2x}\cdot\left(2^{\sqrt{x}}(\ln 2)\cdot\frac{1}{2\sqrt{x}} + \frac{10\,tgx}{\cos^2x}\right)$$

Mnożymy obie strony równości przez $f(x)$:

$$f'(x) = \underbrace{(2^{\sqrt{x}} + 5tg^2x)^{\sin x}}_{=f(x)}\cdot\Big\{\left[(\cos x)\cdot\ln(2^{\sqrt{x}} + 5tg^2x)\right]$$

$$+ (\sin x)\cdot\frac{1}{2^{\sqrt{x}} + 5tg^2x}\left(2^{\sqrt{x}}\cdot\frac{\ln 2}{2\sqrt{x}} + \frac{10\,tgx}{\cos^2x}\right)\Big\}.$$

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