





# Pochodne

przykładowe zadania

na kartkówkę

$$1. f(x) = \sqrt[3]{x} \cdot \sin\left(\frac{x^3+1}{2x+\ln(5x)}\right)$$

$$2. f(x) = \cos\left(\arctg\left(\sqrt{\frac{5}{x^2} + \ln^3 x}\right)\right)$$

$$3. f(x) = \left(2^{\sqrt{x}} + 5 \lg^2 x\right)^{\sin x}$$

Rozwiązanie.

Ad 1.

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \cdot \sin\left(\frac{x^3+1}{2x+\ln(5x)}\right) +$$

$$+ \sqrt[3]{x} \cdot \left[\cos\left(\frac{x^3+1}{2x+\ln(5x)}\right)\right] \cdot \frac{3x^2 \cdot (2x+\ln(5x)) - (x^3+1) \cdot \left(2 + \frac{1}{5x} \cdot 5\right)}{(2x+\ln(5x))^2}$$



Ad 2.

$$f'(x) = - \left[ \sin \left( \arctg \left( \sqrt{\frac{5}{x^2} + \ln^3 x} \right) \right) \right] \cdot$$

$$\cdot \frac{1}{1 + \left( \sqrt{\frac{5}{x^2} + \ln^3 x} \right)^2}$$

$$\cdot \frac{1}{2 \sqrt{\frac{5}{x^2} + \ln^3 x}} \cdot \left( -\frac{10}{x^3} + 3(\ln^2 x) \cdot \frac{1}{x} \right)$$

Ad 3.

$$f(x) = \left( 2^{\sqrt{x}} + 5 \operatorname{tg}^2 x \right)^{\sin x} \quad | \quad \ln(\dots)$$

$$\ln(f(x)) = \ln \left( 2^{\sqrt{x}} + 5 \operatorname{tg}^2 x \right)^{\sin x}$$

$$\ln(f(x)) = \sin x \cdot \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) \quad | \quad (\dots)'$$

$$\left( \ln(f(x)) \right)' = \left( \sin x \cdot \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) \right)'$$

1. LOCZYN  $\rightarrow (fg)' = f'g + fg'$

$$\frac{1}{f(x)} \cdot f'(x) = (\sin x)' \cdot \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) + (\sin x) \cdot \left( \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) \right)'$$



$$\frac{f'(x)}{f(x)} = (\cos x) \cdot \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) +$$

$$+ (\sin x) \cdot \frac{1}{2^{\sqrt{x}} + 5 \operatorname{tg}^2 x} \cdot (2^{\sqrt{x}} + 5 \operatorname{tg}^2 x)'$$

$$(a^{f(x)})' = a^{f(x)} \cdot (\ln a) \cdot f'(x)$$

$$\operatorname{tg}^2 x = (\operatorname{tg} x)^2$$

$$((\operatorname{tg} x)^2)' = 2(\operatorname{tg} x) \cdot (\operatorname{tg} x)' = \dots$$

$$\frac{f'(x)}{f(x)} = (\cos x) \cdot \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) +$$

$$+ (\sin x) \cdot \frac{1}{2^{\sqrt{x}} + 5 \operatorname{tg}^2 x} \cdot \left( 2^{\sqrt{x}} \cdot (\ln 2) \cdot \frac{1}{2\sqrt{x}} + \frac{10 \operatorname{tg} x}{\cos^2 x} \right)$$

Możemy obie strony pomnożyć przez  $f(x)$ :

$$f'(x) = \underbrace{(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x)}_{= f(x)}^{\sin x} \cdot \left[ (\cos x) \cdot \ln(2^{\sqrt{x}} + 5 \operatorname{tg}^2 x) \right]$$

$$+ (\sin x) \cdot \frac{1}{2^{\sqrt{x}} + 5 \operatorname{tg}^2 x} \left( 2^{\sqrt{x}} \cdot \frac{\ln 2}{2\sqrt{x}} + \frac{10 \operatorname{tg} x}{\cos^2 x} \right)$$





Wieroyel  
Smit